

Matematika sa Makabagong Daigdig

Matematika sa Ating Mundo

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Matematika sa ating mundo

Ang matematika ay isang kapaki-pakinabang na paraan ng pag-iisip tungkol sa kalikásan at sa ating mundo.

Inaasahang matutuhan

- ▶ Tumukoy ng mga disenyo sa kalikásan at mga regularidad sa mundo.
- ▶ Ipaliwanag ang kahalagahan ng matematika sa buhay.
- ▶ Suriin ang kalikásan ng matematika, kung ano ito, paano ito ipinakikita, nirerepresenta, at ginagamit.
- ▶ Ipakita ang pagpapahalaga sa matematika bilang gawaing pantao.

Kalikásan sa pamamagitan ng mga numero

Isang maikling pelikula ni Cristóbal Vila, 2010

Orihinal (3:44): <https://vimeo.com/9953368>

Alternatibong *soundtrack* (4:04): <https://vimeo.com/29379521>

Fibonacci *sequence*

(Vila, 2016)

$$0+1=1$$

$$0\ 1+1=2$$

$$0\ 1\ 1+2=3$$

$$0\ 1\ 1\ 2+3=5$$

$$0\ 1\ 1\ 2\ 3+5=8$$

$$0\ 1\ 1\ 2\ 3\ 5+8=13$$

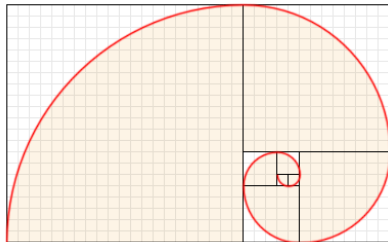
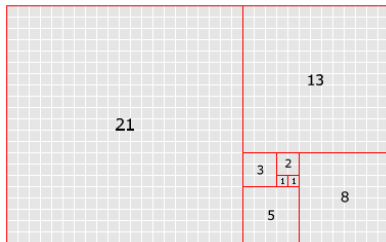
etc

0 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597 2584 4181

Ang Fibonacci *sequence* ay isang walang katapusang pagkakasunod-sunod ng mga natural na numero kung saan ang unang halaga ay 0, ang susunod ay 1 at, mula doon, ang bawat halaga ay nakuha sa pamamagitan ng pagdaragdag ng nakaraang dalawa.

Fibonacci *spiral*

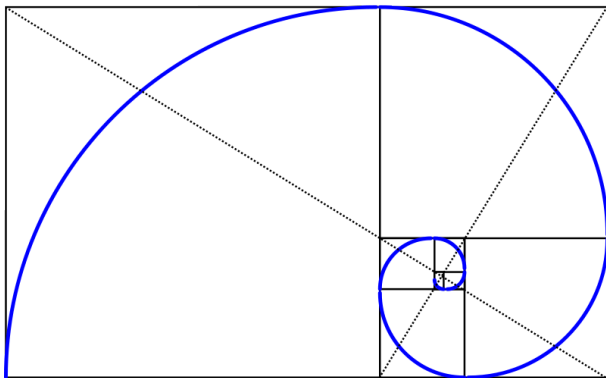
(Vila, 2016)



Ang magkabilang sulok ng mga parisukat sa Fibonacci *tiling* ay konektado sa pamamagitan ng pabilog na mga arko.

Golden spiral

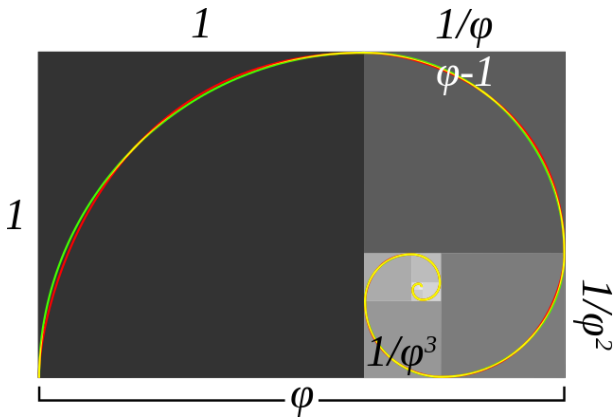
(Golden spiral in rectangles, 2008)



$r = \varphi^{2\theta/\pi}$; ang θ ay nasa *radians* at ang $\varphi = \frac{1+\sqrt{5}}{2}$ ay ang *golden ratio*

Paghahambing ng mga *spiral*

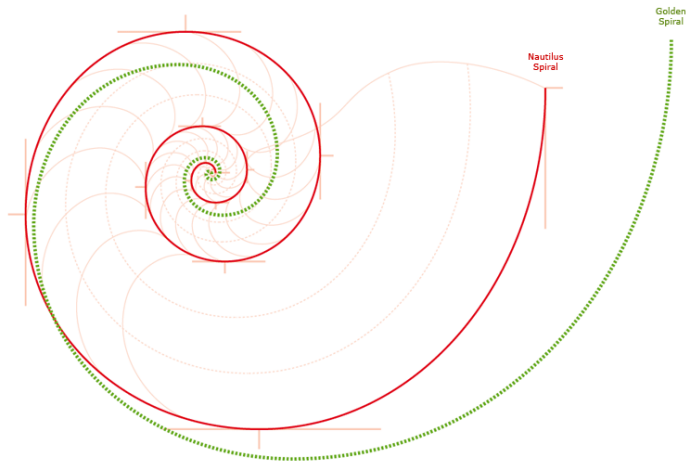
(Approximate and true Golden Spirals, 2009)



Berde ang mga isang-kapat na bilog, pula ang *golden spiral*, dilaw ang mga pagsanib.

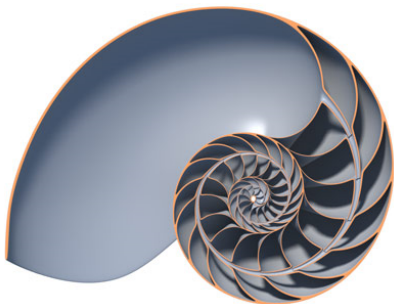
Nautilus *spiral*

(Vila, 2016)



Pagkakamali ni Vila

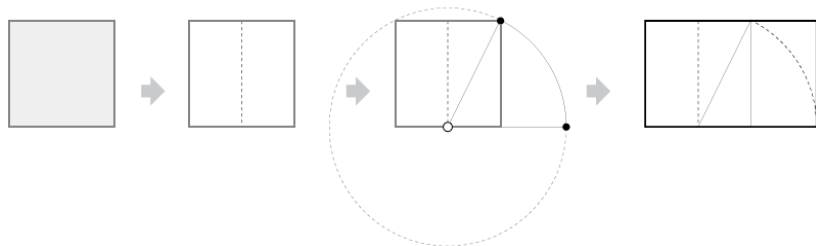
(Vila, 2016)



Inaamin ni Vila na nagkamali siya sa animasyon para sa Nautilus *shell*. (Hindi ito Fibonacci *spiral* o *golden spiral*.)

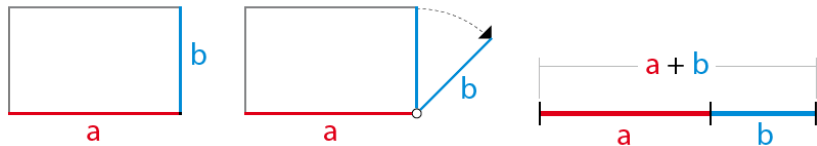
Golden rectangle

(Vila, 2016)



Golden ratio

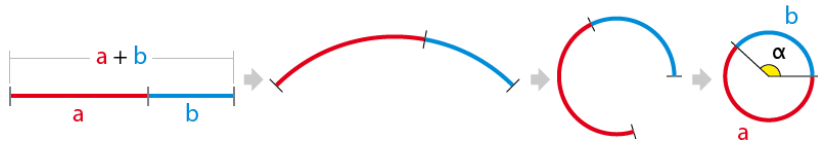
(Vila, 2016)



$$\frac{a}{b} = \frac{a+b}{a} = \varphi \text{ (Phi)} = 1.61803399\dots$$

Golden angle

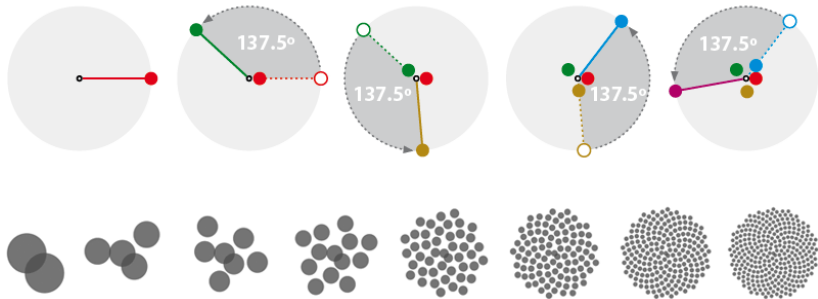
(Vila, 2016)



$$\frac{a}{b} = \frac{a+b}{a} = \varphi \text{ (Phi)} = 1.61803399... \rightarrow \alpha = 137.507764^\circ... \sim 137.5^\circ$$

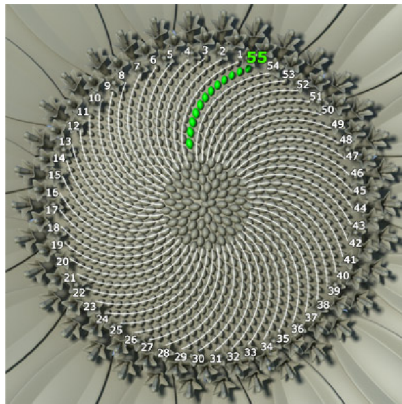
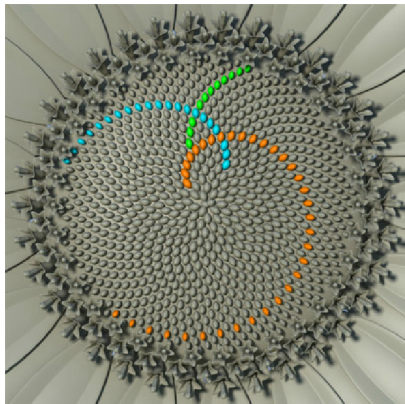
Golden angle at ang pag-aayos ng binhi ng mirasol

(Vila, 2016)



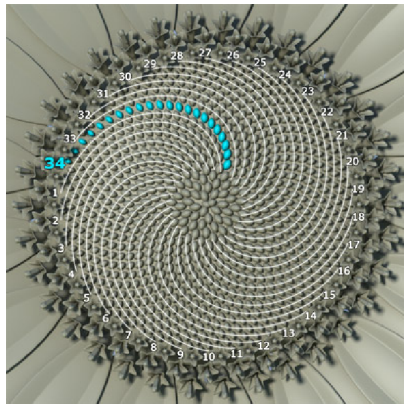
Fibonacci *numbers* at ang pag-aayos ng binhi ng mirasol

(Vila, 2016)

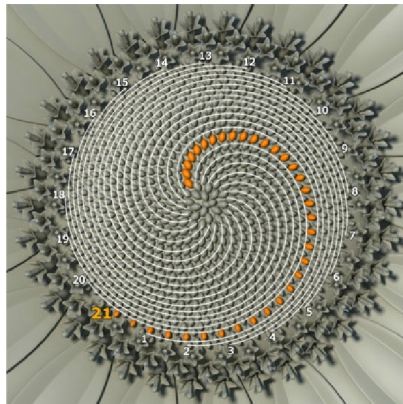


0 1 1 2 3 5 8 13 21 34 **55** 89 144

Fibonacci *numbers* at ang pag-aayos ng binhi ng mirasol (pagpapatuloy) (Vila, 2016)



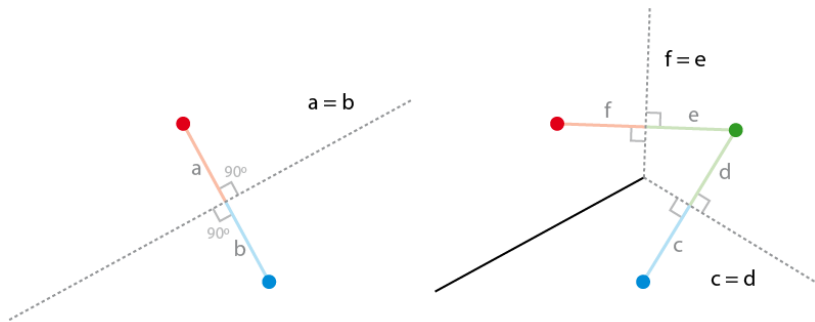
0 0 1 1 2 3 5 8 13 21 **34** 55 89 144



0 0 1 1 2 3 5 8 13 **21** 34 55 89 144

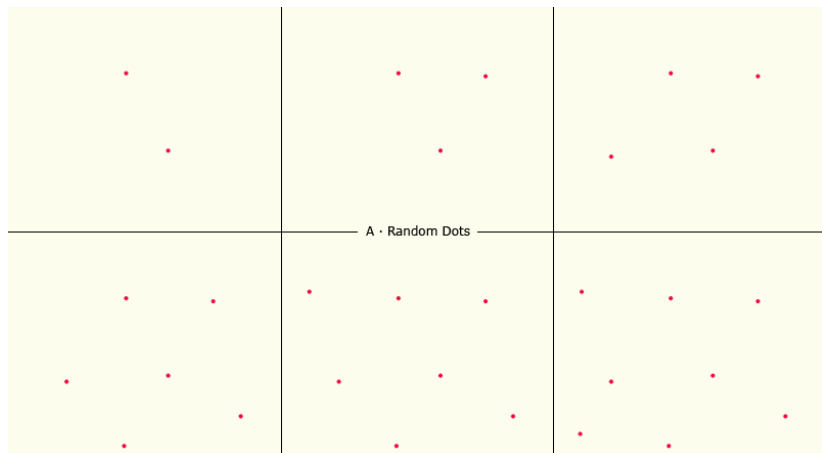
Delaunay *triangulation* at Voronoi *diagram*

(Vila, 2016)



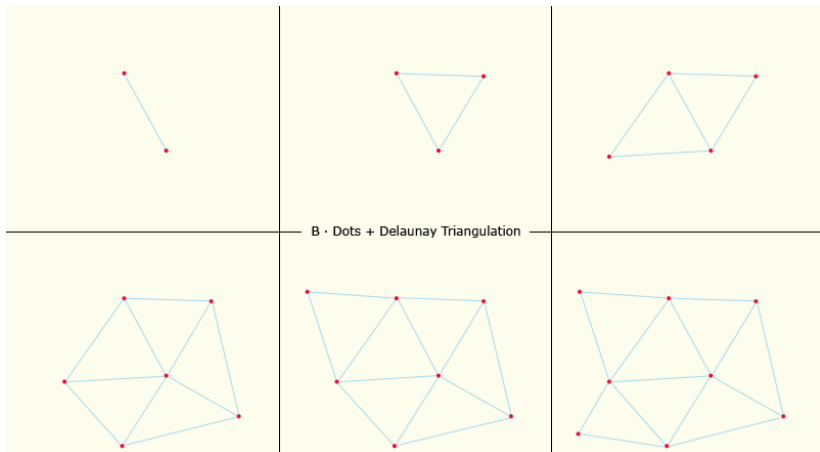
Delaunay *triangulation* at Voronoi *diagram*

(pagpapatuloy) (Vila, 2016)



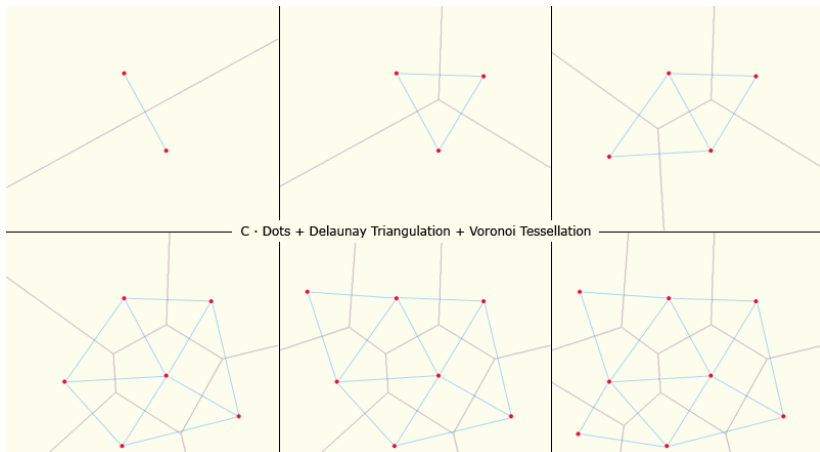
Delaunay *triangulation* at Voronoi *diagram*

(pagpapatuloy) (Vila, 2016)



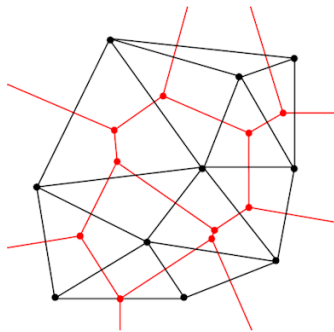
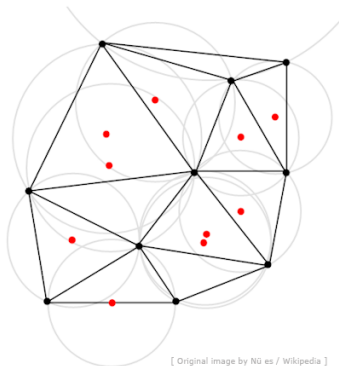
Delaunay *triangulation* at Voronoi *diagram*

(pagpapatuloy) (Vila, 2016)



Delaunay *condition*

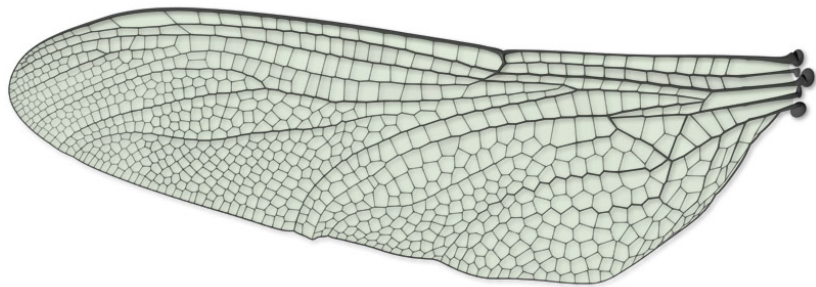
(Vila, 2016; Delaunay triangulation, 2017)



Ang isang Delaunay *triangulation* para sa isang hanay ng mga punto sa isang patag na ibabaw ay isang *triangulation* na walang punto sa loob ng *circumcircle* ng anumang tatsulok.

Pakpak ng tutubi

(Vila, 2016)



Relasyon ng *golden ratio* at *Fibonacci sequence*

(Vila, 2016; Golden ratio, 2017)

$$1/1 = 1$$

$$2/1 = 2$$

$$3/2 = 1.5$$

$$5/3 = 1.666666666666$$

$$8/5 = 1.6$$

$$13/8 = 1.625$$

$$21/13 = 1.61538461538$$

$$34/21 = 1.61904761905$$

$$55/34 = 1.61764705882$$

$$89/55 = 1.61818181818$$

—

$$\mathbf{\text{Phi}} = \mathbf{1.6180339887\dots}$$

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2} \text{ for } n > 1, n \in \mathbb{Z}$$

$$F_n = \frac{\varphi^n - (1 - \varphi)^n}{\sqrt{5}} = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}}$$

$$\lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n} = \varphi$$

Ang matematika ay isang kapaki-pakinabang na paraan ng pag-iisip tungkol sa kalikásan

(Stewart, 1995, p. 19)

Whatever the reasons, mathematics definitely is a useful way to think about nature. What do we want it to tell us about the patterns we observe? There are many answers. We want to understand how they happen; to understand why they happen, which is different; to organize the underlying patterns and regularities in the most satisfying way; to predict how nature will behave; to control nature for our own ends; and to make practical use of what we have learned about our world. Mathematics helps us to do all these things, and often it is indispensable.

Takdang aralin

Basahin si Stewart (1995) at maging handa upang sagutin ang mga sumusunod na tanong sa talakayan.

- ▶ Aling pangungusap o talata sa aklat ang paborito mo? Bakit?
- ▶ Mayroon bang pahayag o pananaw sa aklat na hindi ka sumasang-ayon?
- ▶ Paano mo ibubuod ang bawat isa sa siyam na kabanata (sa isa, dalawa, o tatlong pangungusap bawat kabanata)?
- ▶ Paano ni Stewart ipinagkakaiba ang panlabas na aspeto ng matematika at ang panloob na aspeto ng matematika?
- ▶ Anong katagang ginagamit ni Stewart upang ilarawan ang kanyang pangarap ng isang epektibong teorya ng matematika ng *form* at ang paglitaw ng *pattern*?

Takdang aralin

Gamit si Stewart (1995) bilang sanggunian, magsulat ng isang sanaysay na may 250 hanggang 350 na salita na sinasagot ang isa sa mga sumusunod na tanong.

- ▶ Paano ipinaliwanag ni Stewart kung bakit lumilitaw ang mga numero mula sa Fibonacci *series* kapag binibilang ang ilang mga tampok sa mga halaman?
- ▶ Nagkaroon ng argumento si Leonhard Euler kay Daniel Bernoulli dahil ang kanilang mga solusyon sa *one-dimensional wave equation* ay naiiba. Paano ipinaliwanag ni Stewart ang resolusyon ng argumento?
- ▶ Paano ginamit ni Stewart ang konsepto ni Poincaré na *phase space* upang ipaliwanag kung bakit ang pagtaas ng tubig (*tides*) ay maaaring mahulaan ngunit ang panahon (*weather*) ay hindi?
- ▶ Paano ginamit ni Stewart ang mga *coupled oscillator network* sa pagmomodelo ng paglakad ng hayop?

Ang matematika ba ay natutuklasan o naiimbento?

Isang TED-Ed *Original lesson* ni Jeff Dekofsky, 2014

(5:11): https://youtu.be/X_xR5Kes4Rs

Tingnan rin ang <http://ed.ted.com/lessons/is-math-discovered-or-invented-jeff-dekofsky>.

Pilosopiya ng matematika

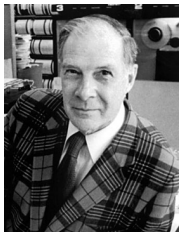
(Philosophy of Mathematics, 2017)

“*Mathematical realism* [...] holds that mathematical entities exist independently of the human mind. Thus humans do not invent mathematics, but rather discover it [...].” Isang anyo ng *mathematical realism* ay *Platonism*.

“*Mathematical anti-realism* generally holds that mathematical statements have truth-values, but that they do not do so by corresponding to a special realm of immaterial or non-empirical entities.” Isang anyo ng *mathematical anti-realism* ay *formalism*.

Platonism

“*Mathematical Platonism* is the form of realism that suggests that mathematical entities are abstract, have no spatiotemporal or causal properties, and are eternal and unchanging.” (Philosophy of Mathematics, 2017)



(Richard Hamming, 2013)

“Very few of us in our saner moments believe that the particular postulates that some logicians have dreamed up create the numbers—no, most of us believe that the real numbers are simply there and that it has been an interesting, amusing, and important game to try to find a nice set of postulates to account for them.” (Hamming, 1980, p. 85)

Formalism

“*Formalism* holds that mathematical statements may be thought of as statements about the consequences of certain string manipulation rules.” (Philosophy of Mathematics, 2017)



(David Hilbert, 2017)

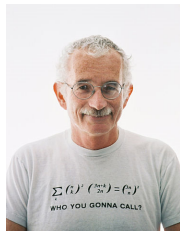
“Mathematics, according to David Hilbert (1862-), is a game played according to certain simple rules with meaningless marks on paper.” (Stabler, 1935, p. 24)

Ultrafinitism

“*[C]onstructivism* involves the regulative principle that only mathematical entities which can be explicitly constructed in a certain sense should be admitted to mathematical discourse. [...]

Finitism is an extreme form of constructivism, according to which a mathematical object does not exist unless it can be constructed from natural numbers in a finite number of steps. [...]

Ultrafinitism is an even more extreme version of finitism, which rejects not only infinities but finite quantities that cannot feasibly be constructed with available resources.” (Philosophy of Mathematics, 2017)



(Doron Zeilberger, 2007)

“What is completely meaningless is any kind of *infinite*, actual or potential. So I deny even the existence of the Peano axiom that every integer has a successor. [...]

The phrase ‘for *all* positive integers’ is meaningless. [...]

Similarly, Euclid’s statement: ‘There are infinitely many primes’ is meaningless.” (Zeilberger, 2001, p. 5)

Nakabubuo at di-nakabubuo na patunay

Teorama

May mga irrational na numero na a at b na ang a^b ay rational.

Di-nakabubuo na patunay.

Isalang-alang ang $\sqrt{2}^{\sqrt{2}}$. Kung ito ay *rational*, kumpleto na ang patunay. Kung hindi ito *rational*, kunin ang $a = \sqrt{2}^{\sqrt{2}}$ at $b = \sqrt{2}$ upang ang $a^b = 2$.

Nakabubuo na patunay (*sketch*).

Kunin ang $a = \sqrt{2}$ at $b = 2 \log_2 3$.

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